Degree of a Polynomial

The highest degree of any term in the polynomial.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^4 - 2x + 3$</td>
<td>4</td>
</tr>
<tr>
<td>$4x + x + x^3$</td>
<td>3</td>
</tr>
<tr>
<td>$x^4 - y^4$</td>
<td>4</td>
</tr>
<tr>
<td>$a + 9$</td>
<td>1</td>
</tr>
<tr>
<td>$3x + 2$</td>
<td>1</td>
</tr>
<tr>
<td>$3$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Polynomials: Definitions / Evaluation**

By now, you should be familiar with variables and exponents, and you may have dealt with expressions like $3x^4$ or $6x$. Polynomials are sums of these "variables and exponents" expressions. Each piece of the polynomial, each part that is being added, is called a "term". Polynomial terms have variables which are raised to whole-number exponents (or else the terms are just plain numbers); there are no square roots of variables, no fractional powers, and no variables in the denominator of any fractions. Here are some examples:

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Is a Polynomial Term?</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6x^{-2}$</td>
<td>No</td>
<td>Because the variable has a negative exponent.</td>
</tr>
<tr>
<td>$1/x^2$</td>
<td>No</td>
<td>Because the variable is in the denominator.</td>
</tr>
<tr>
<td>$\sqrt{x}$</td>
<td>No</td>
<td>Because the variable is inside a radical.</td>
</tr>
<tr>
<td>$4x^2$</td>
<td>Yes</td>
<td>Because it obeys all the rules.</td>
</tr>
</tbody>
</table>

Here is a typical polynomial:
Notice the exponents on the terms. The first term has an exponent of 2; the second term has an "understood" exponent of 1; and the last term doesn't have any variable at all. Polynomials are usually written this way, with the terms written in "decreasing" order; that is, with the largest exponent first, the next highest next, and so forth, until you get down to the plain old number.

Any term that doesn't have a variable in it is called a "constant" term because, no matter what value you may put in for the variable $x$, that constant term will never change. In the picture above, no matter what $x$ might be, 7 will always be just 7.

The first term in the polynomial, when it is written in decreasing order, is also the term with the biggest exponent, and is called the "leading term".

The exponent on a term tells you the "degree" of the term. For instance, the leading term in the above polynomial is a "second-degree term" or "a term of degree two". The second term is a "first degree" term. The degree of the leading term tells you the degree of the whole polynomial; the polynomial above is a "second-degree polynomial". Here are a couple more examples:

- **Give the degree of the following polynomial:** $2x^5 - 5x^3 - 10x + 9$
  
  This polynomial has four terms, including a fifth-degree term, a third-degree term, a first-degree term, and a constant term.

  **This is a fifth-degree polynomial.**

- **Give the degree of the following polynomial:** $7x^4 + 6x^2 + x$
  
  This polynomial has three terms, including a fourth-degree term, a second-degree term, and a first-degree term. There is no constant term.

  **This is a fourth-degree polynomial.**

When a term contains both a number and a variable part, the number part is called the "coefficient". The coefficient on the leading term is called the "leading" coefficient.
In the above example, the coefficient of the leading term is 4; the coefficient of the second term is 3; the constant term doesn't have a coefficient.

The "poly" in "polynomial" means "many". I suppose, technically, the term "polynomial" should only refer to sums of many terms, but the term is used to refer to anything from one term to the sum of a zillion terms. However, the shorter polynomials do have their own names:

- a one-term polynomial, such as $2x$ or $4x^2$, may also be called a "monomial" ("mono" meaning "one")
- a two-term polynomial, such as $2x + y$ or $x^2 - 4$, may also be called a "binomial" ("bi" meaning "two")
- a three-term polynomial, such as $2x + y + z$ or $x^3 + 4x^2 - 4$, may also be called a "trinomial" ("tri" meaning "three")

I don't know if there are names for polynomials with a greater numbers of terms; I've never heard of any names other than what I've listed.

Polynomials are also sometimes named for their degree:

- a second-degree polynomial, such as $4x^2$, $x^2 - 9$, or $ax^2 + bx + c$, is also called a "quadratic"
- a third-degree polynomial, such as $-6x^3$ or $x^3 - 27$, is also called a "cubic"
- a fourth-degree polynomial, such as $x^4$ or $2x^4 - 3x^2 + 9$, is sometimes called a "quartic"
- a fifth-degree polynomial, such as $2x^5$ or $x^5 - 4x^3 - x + 7$, is sometimes called a "quintic"

There are names for some of the higher degrees, but I've never heard of any names being used other than the ones I've listed.

By the way, yes, "quad" generally refers to "four", as when an ATV is referred to as a "quad bike". For polynomials, however, the "quad" from "quadratic" is derived from the Latin for "making square". As in, if you multiply length by width (of, say, a room) to find the area in "square" units, the units will be raised to the second power. The area of a room that is 6 meters by 8 meters is $48 \text{ m}^2$. So the "quad" refers to the four corners of a square, from the geometrical origins of parabolas and early polynomial

**Evaluation**

"Evaluating" a polynomial is the same as evaluating anything else: you plug in the given value of $x$, and figure out what $y$ is supposed to be. For instance:
• **Evaluate** \(2x^2 - x^2 - 4x + 2\) at \(x = -3\)

I need to plug in "\(-3\)" for the "\(x\)". Remembering to be careful with my parentheses and the **negatives**:

\[
2(-3)^3 - (-3)^2 - 4(-3) + 2 \\
= 2(-27) - (9) + 12 + 2 \\
= -54 - 9 + 14 \\
= -63 + 14 \\
= -49
\]

Always remember to be careful with the minus signs!

**Polynomials: Combining "Like Terms"** *(page 2 of 2)*

Sections: [Polynomial basics](#), Combining "like terms"

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Probably the most common thing you will be doing with polynomials is "combining like terms". This is the process of adding together whatever terms you can, but not overdoing it by trying to add together terms that can't actually be combined. Terms can be combined ONLY IF they have the exact same variable part. Here is a rundown of what's what:

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x) and 3</td>
<td>NOT like terms</td>
<td>The second term has no variable</td>
</tr>
<tr>
<td>(4x) and (3y)</td>
<td>NOT like terms</td>
<td>The second term now has a variable, but it doesn't match the variable of the first term</td>
</tr>
<tr>
<td>(4x) and (3x^2)</td>
<td>NOT like terms</td>
<td>The second term now has the same variable, but the degree is different</td>
</tr>
<tr>
<td>(4x) and (3x)</td>
<td>LIKE TERMS</td>
<td>Now the variables match and the degrees match</td>
</tr>
</tbody>
</table>

Once you have determined that two terms are indeed "like" terms and can indeed therefore be combined, you can then deal with them in a manner similar to what you did in grammar school. When you were first learning to add, you would do "five apples and six apples is eleven apples". You have since learned that, as they say, "you can't add apples and oranges". That is, "five apples and six oranges" is just a big pile of fruit; it isn't something like "eleven applanges". Combining like terms works much the same way.

• **Simplify** \(3x + 4x\)

These are like terms since they have the same variable part, so I can combine the terms: three \(x\)'s and four \(x\)'s makes seven \(x\)'s:
\[3x + 4x = 7x\]

- **Simplify \(2x^2 + 3x - 4 - x^2 + x + 9\)**

  It is often best to group like terms together first, and then simplify:

  \[
  2x^2 + 3x - 4 - x^2 + x + 9 \\
  = (2x^2 - x^2) + (3x + x) + (-4 + 9) \\
  = x^2 + 4x + 5
  \]

  In the second line, many students find it helpful to write in the understood coefficient of 1 in front of variable expressions with no written coefficient, as is shown in red below:

  \[
  (2x^2 - x^2) + (3x + x) + (-4 + 9) \\
  = (2x^2 - 1x^2) + (3x + 1x) + (-4 + 9) \\
  = 1x^2 + 4x + 5
  \]

  It is not required that the understood 1 be written in when simplifying expressions like this, but many students find this technique to be very helpful. Whatever method helps you consistently complete the simplification is the method you should use.

- **Simplify \(10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6\)**

  \[
  10x^3 - 14x^2 + 3x - 4x^3 + 4x - 6 \\
  = (10x^3 - 4x^3) + (-14x^2) + (3x + 4x) - 6 \\
  = 6x^3 - 14x^2 + 7x - 6
  \]

  Warning: When moving the terms around, remember that the terms' signs move with them. Don't mess yourself up by leaving orphaned “plus” and “minus” signs behind.

- **Simplify \(25 - (x + 3 - x^2)\)**

  The first thing I need to do is take the **negative** through the **parentheses**:

  \[
  25 - (x + 3 - x^2) \\
  = 25 - x - 3 + x^2 \\
  = x^2 - x + 25 - 3 \\
  = x^2 - x + 22
  \]

  If it helps you to keep track of the negative sign, put the understood 1 in front of the parentheses:

  \[
  25 - (x + 3 - x^2) \\
  = 25 - 1(x + 3 - x^2) \\
  = 25 - 1x - 3 + 1x^2 \\
  = 1x^2 - 1x + 25 - 3 \\
  = 1x^2 - 1x + 22 \\
  = x^2 - 1x + 22
  \]
While the first format (without the 1’s being written in) is the more “standard” format, either format should be acceptable (but check with your instructor). You should use the format that works most successfully for you.

- **Simplify** \( x + 2(x – [3x – 8] + 3) \)

Warning: This is the kind of problem that us math teachers love to put on tests (yes, we’re cruel people), so you should expect to need to be able to do this.

This is just an order of operations problem with a variable in it. If I work carefully from the inside out, paying careful attention to my “minus” signs, then I should be fine:

\[
x + 2(x – [3x – 8] + 3) \\
= x + 2(x – 1[3x – 8] + 3) \\
= x + 2(x – 3x + 8 + 3) \\
= x + 2(–2x + 11) \\
= x – 4x + 22 \\
= –3x + 22
\]

- **Simplify** \( [(6x – 8) – 2x] – [(12x – 7) – (4x – 5)] \)

I’ll work from the inside out:

\[
[(6x – 8) – 2x] – [(12x – 7) – (4x – 5)] \\
= 4x – 8 – 8x + 2 \\
= –4x – 6
\]

- **Simplify** \( –4y – [3x + (3y – 2x + 2y – 7) – 4x + 5] \)

\[
–4y – [3x + (3y – 2x + 2y – 7) – 4x + 5] \\
= –4y – [3x + (3y – 2x + 2y – 7) – 4x + 5] \\
= –4y – [3x + (–2x + 5y – 7) – 4x + 5] \\
= –4y – [3x – 2x + 5y – 7 – 4x + 5] \\
= –4y – [3x – 2x + 5y – 7 + 5] \\
= –4y – [–3x + 5y – 2] \\
= –4y + 3x – 5y + 2 \\
= 3x – 4y – 5y + 2 \\
= 3x – 9y + 2
\]

If you think you need more practice with this last type of problem (with all the brackets and the negatives and the parentheses, then review the "Simplifying with Parentheses" lesson.)

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Warning: Don’t get careless and confuse multiplication and addition. This may sound like a silly thing to say, but it is the most commonly-made mistake (after messing up the order of operations):
\[(x)(x) = x^2 \quad \text{(multiplication)}\]

\[x + x = 2x \quad \text{(addition)}\]

"\(x^2\) DOES NOT EQUAL "\(2x\)"

So if you have something like \(x^3 + x^2\), DO NOT try to say that this somehow equals something like \(x^5\) or \(5x\). If you have something like \(2x + x\), DO NOT say that this somehow equals something like \(2x^2\).